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Homotopy Perturbation Method for the Solution of Partial Differential Equations with Different Boundary Conditions

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Abstract

The Homotopy Perturbation Method is among the most effective analytical methods for solving nonlinear partial differential equations with various boundary conditions. In this study, the strengths and weaknesses of the HPM will be presented, as well as the use of HPM combined with other techniques, like the Laplace transform and the Sumudu transform, in solving nonlinear partial differential equations. The study will also compare the accuracy, convergence, and applicability of the Homotopy Perturbation Method with the Adomian Decomposition Method and the Variational Iteration Method based on the literature review. The study reveals that HPM provides a high convergence rate and accuracy and can handle different boundary conditions, making it more effective than other numerical approaches. The study finds that other advancements in the HPM, including the Delta HP Method, will enhance the efficacy of solutions of nonlinear partial differential equations in various fields. As much as the Homotopy Perturbation Method is efficient in providing solutions to nonlinear partial differential equations, challenges have been noted while using the method for equations with high-frequency solutions as well as non-elementary boundary conditions. Further improvement of the Homotopy Perturbation Method is recommended for solving nonlinear partial differential equations, and this method can be used in combination with other methods to expand the range of its application.

Keywords: Homotopy perturbation method, Homotopy perturbation theory, Partial differential equations, Numerical solutions

1. Introduction

The Homotopy Perturbation Method (HPM) is among the widely used analytical techniques for solving partial differential equations (PDEs), especially when subjected to varying boundary conditions. Formulated by Ji Huan in 1999, this method merges the homotopy concepts in topology and conventional techniques of perturbation that help transform compounding nonlinear problems into iterative solvable ones.

This work delivers an extensive assessment regarding the HPM methodology for resolving nonlinear PDEs with diverse boundary conditions. This research develops an organized technique for HPM boundary condition applications while evaluating performance and comparing results to conventional analytical techniques. This study uses the following research questions to direct its investigation (1):

The novelty in this study stems from its extensive research of HPM, together with other methods used for handling nonlinear PDEs with different boundary conditions. The study integrates HPM with different analytical methods for improved use in solving complex nonlinear PDEs. Researchers will achieve a valuable understanding of analytical techniques that solve nonlinear problems through contrastive evaluations between HPM and other existing methods.

2. Literature Review

The HPM has emerged as a useful tool for obtaining analytical and numerical solutions for PDEs under various boundary conditions. It combines the above-discussed perturbation methods with the concept of homotopy borrowed from topology, which helps in simplifying a complex problem through deformation. As such, HPM has been widely accepted for analyzing both linear and nonlinear equations by several researchers in

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applied mathematics and engineering disciplines.

2.1 Homotopy Perturbation Theory

HPM can be defined as a combination of homotopy of topology and perturbation techniques. In the case of homotopy, the transformation between two functions is continuous, while in the perturbation methods that are of a more general nature, a solution is expressed as a power series in terms of a small parameter (2). HPM reduces complex problems into simpler forms that can be solved easily by adding a value of embedding parameter p, which ranges from 0 to 1. This leads the problem to transform into a simpler version

The HPM consists of constructing a homotopy that connects the given system of the nonlinear type with the linear one. There is also a solution in terms of series expansion with respect to the embedding parameter (3). An approximate solution can be obtained by taking a finite number of terms from this series to the required number of terms. This is because the series normally converge rapidly, and therefore, it is easier to obtain accurate results by taking a small number of terms.

2.2 Application Advantages of HPM in Solving PDEs

HPM has been used in solving many types of PDEs, such as the Burgers equation, heat equations, and equations of fluid dynamics. For example, Nourazar et al. recorded the applicability of HPM in obtaining the exact solutions to the Burgers-Huxley equation using the overall power of the technique in providing accurate solutions using fewer computational steps In the same year (3), used HPM to study viscous fluid flow from an exponentially shrinking sheet, which also illustrates the applicability of HPM in handling boundary conditions.

However, HPM has been combined with other mathematical methods in order to expand its application. For instance, researchers have applied a hybrid model of HPM with the Laplace transform to solve fractional differential equations, which offers a stable model for handling systems involving fractional dynamics. This is due to the fact that LT-HPM is effective in solving linear and nonlinear PDEs with both initial and boundary conditions. At times, the Laplace transform can be applied to change the governing equations into algebraic equations or differential equations, which can be easily disturbed around the approximate solution before applying HPM (4). Therefore, the new formulation provides an excellent method that has been implemented as the best strategy for approximating the solution, especially in fractional order of PDEs, viscoelasticity, and diffusion equations where HPMS has had a problem with slow convergence in other formulations.

Another crucial factor that needs to be addressed in solving PDEs is that of boundary conditions. It has also been demonstrated that HPM can handle a wide range of boundary conditions, such as Dirichlet and Neumann, and even a combination of both. For example, Kumar et al. applied HPM to compute analytical solutions of the telegraph equations of the different boundary problems, confirming the effectiveness of the proposed method in providing accurate solutions based on the problem under consideration. It is for this reason that this quality is desirable since other applications often involve boundary conditions, which can have a resource effect on the solution (5).

In addition, it has been seen in the previous sections that HPM has a fast convergence, and hence, a good approximate solution can be obtained with a small number of terms of the series expansion. This is useful in situations where hard resources typing computer, are limited HPM is also flexible enough to construct solutions to different types of problems involving various boundary conditions.

2.3 Comparison with Other Methods

The literature review findings have identified that HPM has a clearer advantage over ADM and VIM in terms of efficiency. For instance, from the study by Otoo et al. (2022), it was evident that the HPM had better results for the nonlinear differential equations describing diarrheal transmission than the ADM. Moreover, the integration of HPM with the least squares version has also been proposed for precise and convergent solutions of fractional PDEs (6). The algebraic solution of the governing equations also corresponds well with other computational approaches, such as the finite difference method (FDM), finite element method (FEM), and spectral methods. There are some situations in which computer use, geometries, multi-dimensional cases, or cases of discontinuity that HPM cannot solve properly. However, there are some disadvantages associated with numerical methods as well. For instance, the general approaches of numerical methods are based on mesh discretization. This incurs some drawbacks, such as truncation error, and the other is that it requires large computational power.

On the other hand, numerical methods are used to approximate the problems and phenomena related to stochastic oscillations, turbulence, or a large number of other PDEs. As a result, it has been established that it is possible to approximate varying higher accuracy demands with analytical trends that are different from the wholly numeral techniques, in which the adequacy of the computation after some amount of time is definite with regard to orthogonality. Despite this, whenever it is necessary to be precise regarding the precise configurations of the object in question, numerical solutions are preferred. Thus, one should note that HPM is most effective when working with single-variable problems when the field problem may be solved analytically, and when quick approximations are required rather than FDM and FEM.].

2.4 Extensions and Modifications of HPM

Researchers have also given ideas on the extensions and generalizations of HPM for its extended usage. introduced the Elzaki transform HPM for solving time-fractional nonlinear PDEs for the applicability of the method in fractional calculus. Also, the sumudu transform has been implemented and incorporated with the HPM, and it has provided positive results

2.5 Numerical Investigations and Error Analysis

The HPM has also been proven to be efficient by numerical studies. A comparative analysis was made by (6)(7)using a numerical technique with the Haar wavelet series method for HPM to arrive at the fact that the approximations discovered were precise as they had the least absolute error for various intervals. Such a type of numerical analysis is important to show that HPM is reliable and efficient as a newcomer to the world of numerical methods.

3. Method

3.1 Research Design

The research method selected as the most appropriate for this research is a literature-based review, especially concerning the nonlinear PDEs and their resolution using the HPM. The review will focus on the comparison of HPM's solving capability with other perturbation techniques, such as the ADM and the VIM, for nonlinear PDE with different boundary conditions. This is not an algebraic approach to solving mathematical equations because it does not solve the equations directly. However, it is a process of contemplating theory and scrutinizing results obtained from other related studies. The emphasis is made on comparing the performances of each method based on the level of the achieved solution, time efficiency, and how well they perform within a broad spectrum of boundary conditions.

3.2 Data Collection

To make the review exhaustive, both published works contained in scholarly articles will be considered as the sources of data for the analysis of HPM, ADM, and VIM for solving nonlinear PDEs with different boundary conditions. These sources will be retrieved from credible online databases such as PubMed, IEEE Xplore (8), Science Direct, and Google Scholar, among others. The search will focus on the papers that can clearly describe the methods used and the results obtained, building a sound basis for comparison.

Special emphasis will be placed on research works on nonlinear PDEs solved with the help of these perturbation methods and under different boundary conditions. The performance of each method will be captured, and examples will be provided to show the practical application of the methods as well as their effectiveness.

3.3 Procedure

The first aspect of the methodology is to identify other nonlinear PDEs that have been solved using HPM, ADM, and VIM. These will be chosen from published sources depending on the educative nature of their procedures, the level of elaboration of the solution process, and the relevance to the research topic (9). After these studies have been identified, the main points highlighted in the research will be synthesized.

The second step involves narrowing down this data by focusing on three criteria: the convergence rate of the methods, the quality of the solution, and the ability of the methods when it comes to boundary conditions. Each of these factors will play a crucial role in evaluating the efficiency of the proposed methods in solving nonlinear PDEs.

In the third stage, the comparison of theoretical advantages and relevant issues affecting the chosen methods shall be established depending on the literature analysis. This comparison will help to identify how HPM, ADM, and VIM are compared to each other, and this information will enable us to understand when which of the methods is more effective.

3.4 Data Analysis

The analysis will entail a qualitative exploration of the benefits of HPM when applied to solve nonlinear PDEs. Special attention will be paid to the advantages of HPM, including its simplicity and flexibility that allow for its application when solving a large number of nonlinear problems.

The analysis will also consider cases where ADM or VIM would be a better approach than HPM to achieving knowledge management benefits. These will compare the findings of the selected studies, identifying some situations where ADM or VIM is likely to yield better results, some specific nonlinearities, or types of problems at the boundary conditions.

Trends of HPM will also be discussed based on their usage over a period of time. This comprises aspects such as further integration of the method with other methods as well as usage of the technique in various disciplines(10). Knowledge of how HPM has evolved and its application in different fields of study would help set the stage for its usefulness in the present and future.

3.5 Validation

In order to corroborate the reliability of the findings, prior studies with results for the same nonlinear PDEs or similar boundary conditions are considered. This analysis will involve the identification of any similarities or differences from the reported results, and it will offer reference points for establishing general conclusions from the literature review. These results will be then combined in an overview of the comparative performance of HPM, ADM, and VIM when applied to the analysis of nonlinear PDEs.

3.6 Outcome Measures

The result of this methodology will be a theoretical evaluation of HPM in solving nonlinear PDEs, coupled with the comparison of its usefulness to ADM and VIM. The research will help establish whether the benefits of HPM outweigh other techniques and indicate when it is useful to apply other strategies. The findings presented in this work will be beneficial for the researchers and practitioners to know when to use which method and support the development of the solution methods for nonlinear PDEs.

4. Results

4.1 Overview of Reviewed Studies

Selection criteria for the studies to be included in the present study were defined by a good description of technical information, relation to HPM, ADM, and VIM, and the number of applications covered by different researchers. found that the Differential Transformation Method (DTM) is a perfect method and useful for developing the nonlinear heat transfer relation that has similar outcomes as HPM and VIM. In the same context, described the heat transfer characteristics of porous fins analytically through numerical methods and checked the results to demonstrate the accuracy of the technique employed.

Furthermore, the development of HPM was outlined by Huseen (2024), who categorized and generalized the concept of HPM. This augmentation allows the resulting solution method to proceed evenly to some extent in anticipation, as the method is capable of solving a host of nonlinear PDEs. Besides these methodological contributions, the studies also captured the ability to use HPM in mimicking real situations through the modeling of the disease transmission rate and heat distribution. This shows that in these applications, HPM has a functional purpose in the methodologies of analytical or near solutions in nonlinear systems.

4.2 Performance Metrics Comparison

4.2.1 Convergence Speed

Although there are many works reported in the literature for comparing the performance of HPM, ADM, and VIM in terms of convergence, it can be said that the convergence speed of an HPM is much faster than the other two methods for a specific class of non-linear PDE. However, while all of the specified methods help to find the solution to different classes of nonlinear PDEs, they do not have the same rate of convergence. also proved when implementing the test equation of (6) Lagerstrom that a larger number of steps of ADM is necessary to achieve the same level of convergence as in HPM. Therefore, HPM performs well for this type of nonlinear system. In the same way, they also found that HPM achieved the dynamics of transmission of diarrhea to the best decision compared to both ADM and VIM within the least number of iterations within the desired accuracy level.

In contrast, ADM has been determined to converge competitively when it comes to solving some classes of nonlinear PDEs, especially those with exponential-type nonlinearity.The above-investigated evaluation shows that HPM has a faster convergence rate than others for polynomial and rational nonlinearity specifically. A comparison of the convergence speed of the various studies indicates that while VIM can provide various levels of precise solutions, the convergence rate is generally slower than that of HPM and ADM, particularly in complex nonlinear problems.

4.2.2 Solution Accuracy

Studies have been conducted in the past about the accuracy of the solution obtained from HPM, ADM, and VIM, where it was discovered that HPM gave a higher accuracy agreement in different nonlinear PDEs. According to using a non-linear gas dynamic equation case, it was observed that the solutions that were determined for (7) HPM were closer to the exact solution, and this proves a high level of accuracy, taking only a few iterations.

In addition, there is a trend in the accuracy performance with respect to the specified PDEs and boundary conditions. also showed that for the Dirichlet boundary condition, the effectiveness of HPM is constant in terms of HPM steps/ sweeps. On the other hand, ADM is less accurate during the computation of highly nonlinear systems, especially those with mixed boundary conditions, as pointed out. This indicates that though ADM is good for solving specific types of equations, HPM has the added versatility of offering better accuracy when dealing with different boundary conditions.

4.2.3 Ability To Handle Boundary Conditions

Comparative analysis of HPM, ADM, and VIM also covered their ability to provide solutions where possible under various types of boundaries. Surprisingly, this is a great flexibility of HPM in the treatment of the different boundary conditions, making HPM a powerful tool for the approximation of the nonlinear PDE. As an example of its applicability, applied HPM to fuzzy boundary value problems. Furthermore, it has been established that HPM can be used for mixed boundary conditions, for instance, the boundary value conditions, and they can be solved exactly a number of times without having to reformulate the governing equations.

On the other hand, the research revealed that ADM provides less accuracy concerning the number of problems containing mixed boundary conditions, and additional changes are needed for the solutions to converge and give a better prognosis. It has been difficult, particularly when, for a given problem, the optimum solution provided by ADM cannot be extended across the various boundaries. As VIM can solve the differential equations with the determined boundary conditions, pointed out that VIM has some problems with complex boundary conditions, especially if there are nonlinear terms involved.

4.3 Trends in HPM Application

Since its conception in 1999, the HPM has been enhanced and effectively used in numerous scientific and engineering disciplines. The application of HPM with multiple transforms has been increasing over the last few years. This integration has made the solution of nonlinear PDEs even more efficient and effective with the method. For instance, have investigated the application of the Laplace transform (11) with HPM, such as the Laplace Homotopy Perturbation Method (LHPM), to solve conformable PDEs with good accuracy and convergence performance. In the same way, HPM mixed with the Laplace transform has been used in heat transfer, and it, too, provided a reasonably acceptable solution in a reasonable time with a high degree of accuracy

One of the new additions, and worthy of mention, is the δ-HPM, which extends the HPM framework. extended the method to solve general nonlinear PDEs with a higher order of convergence. This restructuring suggests that there is a clear comprehension of the need for more nimble and potent tools that can adequately address increasingly intricate models mathematically. It has especially been utilized in the field of fluid dynamics for turbulent as well as other nonlinear cases, which shows its efficacy and usefulnessNavigation and simulation have used HPM to solve the Navier-Stokes equations for fluid dynamics. have numerically investigated the non-Newtonian nanofluid flow between two concentric cylinders, taking into consideration Joule heating and mixed convection. Additionally, Fatima (2021) explained how HPM is used to(7) solve heat conduction equations, which indicated that HPM is useful in solving heat transfer problems.

Moreover, other analytical approaches to the HPM, along with new hybrids, have also been investigated. The use of HPM with the help of Sumudu transform was discussed where the research was done on time-fractional PDEs. As the work suggested and applied in the work, it is possible to construct more effective solutions to those problems, especially complex ones, which are hard to solve by the traditional way of using the analytical sensitivity method only. HPM has also been used to track disease (12)transmission with regard to epidemiology modeling utilized HPM to research the spread of infectious diseases. This shows that HPM can be applied in the infectious disease field of public health. This shows how closely related HPM is to solving real-life issues that are faced in different organizations.

5. Discussion

5.1 Interpretation of Findings

It is evident from the above studies that the implementation of the HPM can solve PDEs under various boundary conditions. Several factors make it possible for this faster convergence and outperforming conventional methods and approaches like the ADM and the VIM. First, HPM amalgamates the segment known as the homotopy and the perturbation methods, which provide a rigorous approach to dealing with non-linear problems. This made it possible to construct a series solution that converges faster to the actual solution, especially when the nonlinearity is relatively small. However, it must be mentioned that ADM is one of the most popular semi-analytical methods that can be used for (4) solving nonlinear PDEs as well. Unlike HPM, ADM also never employs a linearization approach and gives a recursive series solution. However, it is noticed that in solving highly nonlinear problems, ADM uses the Taylor series and polynomial decomposition Adomian polynomial for the nonlinear terms. However, HPM is less complicated and faster in computation (13) than ADM because the latter has an extra step of decomposition. Moreover, for the assessment of an infinite series, ADM may be slower in convergence compared with the homotopy approach in some instances. However, in the cases of vital nonlinear integral terms, ADM is best applicable since the decomposition method is straightforward in addressing non-polynomial nonlinearity. However, HPM is more efficient as compared to ADM when the computation of simplicity and convergence time is an issue, and all this is attributed to the fact that in HPM, no breaking down of nonlinear terms is required, as is the case with ADM (14).
Also, HPM is better applied in conditions where an explicit homotopy formulation can be advantageous in generating direct analytical approximations, while VIM is better applied where constrained optimization, the system of differential equations is in question since iteration improvement through correction function offers an advantage in such circumstances. The VIM is another semi-approximate method that is based on correction, in which corrections are added iteratively to the construction of the method. This is much more useful, especially when dealing with a system of dependent subsystems or when the problem is an example of constrained optimization. Despite the fact that both HPM procedures are based on the use of iterative frequencies and correction functions, VIM offers much greater flexibility in modifying these values (4). Still, VIM requires more iteration in comparison to HPM and, for this reason, is more computational than HPM while solving many practical, real-life problems. However, in those cases where direct perturbation expansion is desired due to certain topological advantages, explicit construction of a homotopy using HPM is not technically difficult.
It can also be noted that HPM, ADM, and VIM widely differ in terms of approximate solutions' accuracy and computational cost. HPM generally has better convergence rates than ADM and requires fewer applications to achieve the same level of approximation, but for some categories of particular problems, ADM is easier, as it yields higher computational complexity. However, to solve linear and nonlinear equations with VIM, it is required to define the auxiliary functions that can cause difficulties during the solution (6). Thus, the choice of solution method is largely dictated by the specific problem that has to be solved, the nature of the nonlinearity, and the type of boundary conditions.

5.2 Comparison with Existing Literature

It has been established that the HPM can be effectively employed to solve different classes of PDEs. However, since each form of PDE may have more than one boundary condition, the consistency and the kind of conclusions that arrive in different forms of literature are more complicated. Scholars have applied HPM widely because it has been evidenced to help solve linear and nonlinear differential equations with various boundary conditions. However, these are only the general conclusions about the usage of the HPM, and the specific result could be far from the point when particular equations, boundary conditions, or modifications of the HPM are applied.
The computation and analysis of wave phenomenon using HPM is an excellent example of the efficiency of HPM on a large scale for nonlinear bouncers and wave equations. Self and El-Dib (2021) demonstrated that modified HPM can be used to solve axial vibration of the string described

by the Klein-Gordon equation, and the finding yielded reasonable results as expected. Similarly, work on HPM in facilitating the analysis of the transmission rate of COVID-19 showed that the tool was capable of yielding near solutions to complicated models of disease. These studies indicate that HPM always demonstrates a pattern in its operations.

There are variations in the findings that affect them, particularly in relation to the distinctive practices of HPM. While HPM is applicable to solve PDEs to a great extent, other versions have been applied, namely OHAM and HPSTM, to enhance the convergence and accuracy of the solutions. However, it has to be stated that the initial HPM can be considered relatively competitive in terms of both simplicity and computational complexity. Such differences clearly demonstrate that context can seriously affect the assessment of the utility of HPM and its modifications.

The literature also provides certain contingencies concerning the HPM studies where the results were unpredictable. solved the generalized Bratu-type fractional differential equations through HPM and derived the solutions that were not as expected from the behaviors and patterns. Thus, the model necessitates a reevaluation of the assumption. Such cases question whether HPM is effective and call for identifying specific conditions under which HPM would not work or for involving nonlinearity and how such a model would fail to solve it. Such differences are also explained by the fact that HPM is a constantly developing field. Thus, the integration of fractional calculus into HPM is a new and constantly evolving topic that has led to the production of several works that replace certain types of differential equations with fractional differential equations. With these advancements, the challenges involved with the integration and stability of the HPM-derived solutions have escalated because of the need to appreciate the fractional derivative impact.

5.3 Practical Implications

There are pertinent implications of employing HPM to solve PDEs under different boundary conditions. As mentioned before, HPM can address different kinds of boundary conditions, which means that those who use HPM can model various practices in a much closer way to the actual practices, such as fluid dynamics, which, due to the consequence of the fluid boundary conditions, have the ability to influence the dynamics of the physical system even in micro-scales in regions of the flow channels. This HPM feature makes it easier for solutions to arrive at computerized systems that are less solvable with the figure numerical approach (Ullah et al., 2020; Zureigat, 2023).

Similarly, the Sumudu transformed-based HPM known as ST-HPM has gained recognition for being frequently used because it contains the philosophical impact of the original function and also minimizes the probability of time for encountering complexity in a solution. Compared to the Laplace transform, the Sumudu transform does not include an infinite domain, making the process of inversion easier in the case of the former. Researchers have applied ST-HPM to solve time-fractional diffusion equations, heat conduction equations, and reaction-diffusion equations with more accuracy and stability.

Besides integral transforms, HPM has also been combined with different kinds of variations, such as ADM and the homotopy analysis method (HAM). Nonetheless, when it comes to high-order nonlinear PDEs with complex boundary conditions, merely applying HPM as a single entity has certain disadvantages, which led to the development of such hybrids. In addition, VIM-HPM, VIM combined with HPM, has already been shown to increase the efficiency of solutions in problems of flow dynamics and elasticity.

HPM is a system that depends on collaboration between providers and universities and has a number of fruitful effects in different fields of operation. One of the most sensitive areas of utilization is the flow of magnetohydrodynamics (MHD), where generally, the production of a new combination between the fluid motion and magnetic field dominates in engineering for space physics. There is always the option to use HPM to approximate MHD, and such approximations are significant in fusion reactor designs and explanations of geophysical fluids. For these reasons, this technique is preferred in these domains, although there are other numerical techniques, such as numerical grids and iterative solvers, which are standard techniques.

In addition, it has various applications in structural mechanics, electrical engineering, and biological modeling. In structural mechanics applications, HPM has been used to analyze nonlinear vibrations in beams, plates, and shells as a numerical replacement for simulation. This means that HPM has been applied by engineers to estimate the behavior of constructs under diverse loading to perform fast assessments of stability in addition to the dynamic response of elastic members. HPM can be done on all types of equations, including nonlinear forms of diffusion-reaction equations, which are used in semiconductor physical phenomena or charge transport in the semiconductor. As seen from the above discussion, microelectronics, and nanotechnology are rapidly growing fields, and the use of HPM in getting approximate analytical solutions to semiconductor equations has become vital in this area.

In the biological and biomedical engineering areas, the application of HPM has covered the modeling of diffusion phenomena in tissues, drug delivery systems, and reaction-diffusion systems related to population dynamics and disease spread. The nature of these models is nonlinear. Hence, they are difficult to solve using the most common methods. However, HPM is capable of incorporating complicated boundary conditions and/or nonlinear terms to find approximate solutions in epidemiology, biomedical imaging, and tissue engineering. It has also been used in biological and biomedical engineering for simulating tissue permeation, transporting substances in the body, and systems of reactions-diffusions that characterize populations and diseases. These models are also non-linear and could be solved in ways that are complex and difficult to quantify. However, since HPM can handle boundary conditions and nonlinear terms involved, it has proven to be a near-accurate method in epidemiology, biomedical imaging, and tissue engineering.

Nevertheless, it is worth stressing the fact that HPM has negative consequences and shortcomings. One of the weaknesses is the selection of the starting approximation and the construction of a homotopy. Thus, the use of the homotopy continuation method requires the identification of an initial function and the construction of the correct homotopy equation. This is usually done depending on the problem and background of the analyst. At times, there is the possibility of a slow speed of convergence or even a divergence of the solution because the choice may be improper.

Furthermore, HPM provides approximate analytical solutions. However, it can provide the most accurate solution, which is still not an exact one, though it is better than other numerical methods. In such cases, it may be required to go to Padé approximants or homotopy in order to achieve a higher accuracy. This increases the computational costs and makes HPM less simple and easy to use, particularly for practitioners who are well-versed in the said techniques.

It is also important to note that HPM is not efficient when applied to problems that have discontinuities or problems like the solution, which changes with the change in scale. As a result, HPM struggles to handle situations related to a fast oscillating solution, such as shock waves, phase transformations, or contact discontinuities. However, numerical methods can deal with this through adaptive meshing or Discontinuous Galerkin formulations. For such cases, better techniques are possible, such as the finite volume methods (FVM) and spectral methods.

However, in some particular classes of problems, HPM is less efficient, such as when there are high oscillations in the solution or in the nonlinear boundary conditions. However, these restrictions can be mitigated by employing a mixed approach. This may result in changes or preparations of the method that are not optimal for efficiency or repetition.

6. Conclusion

This study presents the efficiency of the HPM in solving nonlinear PDEs with various boundary conditions. HPM shows high speed of convergence, possibility of analytical solution and flexibility which makes it more effective than ADM and VIM. In combination with the Laplace and the Sumudu transforms it widens its area of usage in engineering and physics, frequently in the problems connected with fluids and heat transfer, as well as wave theory.

However, some limitations can be pointed out in the reviewed studies, such as the lack of standardized comparisons for the applied methods, limited numbers of numerical justifications, and the inability to provide detailed analysis of highly non-linear and discontinuous behaviours. The manuscripts which use these constraints might have limitations to their generalizability of the findings which in turn calls for more empirical research.

Future work should consider improving the applicability of HPM for strongly nonlinear equations, the formulation of other differential transformation methods involving HPM, ADM, and VIM, and the applications of such methods in physical sciences, engineering, and technology and computational biology and Quantum mechanics. Since HPM has a wide application area, its interaction with other modern techniques, such as fractional calculus, machine learning, and stochastic analysis, would be an interesting line of work for future study. These novel mixed approaches are presumed to progressively improve their performance in prediction and computation for the development of new scientific fields. Moreover, the computational complexity of these methods can also be optimized, and new, parallelized, or machine learning-based solutions for these methods can be developed further.

Data Availability
Not applicable

Competing Interests
None

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